

# Problems of geometry

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**Abstract:** Using computer-assisted learning or various educational software helps the student learn in a creative way, increases motivation and learning efficiency. The student actively participates in all stages of teaching, learning, assessment, is encouraged to explore the new content, to develop their imagination. I think the computer has become a tool for all those who want to unravel the mysteries of mathematics, a useful tool to students and teachers.

**Keywords:** GeoGebra, Algebra, Geometry, Projects

## Introduction

GeoGebra provides good opportunity for students to work in pairs and talk through the project together. Attractive presentations prepared in advance, not only capture students' attention but also may lessen the immediate cognitive load for educated and educators. In addition to what is traditionally recognized as benefits, a lot of teachers often use real world models. In order to enhance the image mathematics by creating a "halo effect", the proposed efficient space for this will be the GeoGebra platform. The teachers who use GeoGebra must be more specific, more "open minded", willing to allow for experimentation, and give more guidance at the start of any GeoGebra experiment. Dynamic geometry offers opportunities to bring the real world into the classroom, adding visualization, color and animation. This would not be possible in a traditional classroom. This GeoGebra thinking is expected in various topics of the curriculum but, if they are not found there, we shall connect the GeoGebra thinking with topics and other different experiences, in a model of more efficient curricula.

## First problem:

In the triangle  $ABC$ ,  $M$  is the midpoint of  $AC$  and  $Q \in (BM)$ .

If  $QT \parallel AB$ ,  $T \in (BC)$  and the area of  $AQTB$  is equal with  $\frac{5}{16}$  of the

$ABC$  triangle, show that  $BQ = QM$ .

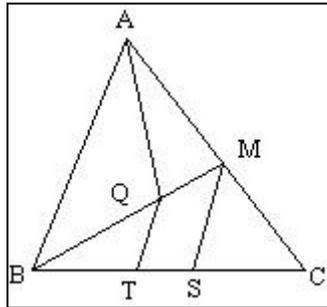
**Solution:**

$$\text{Let } \frac{BQ}{BM} = k \Rightarrow \frac{A_{ABQ}}{A_{ABM}} = k \Rightarrow A_{ABQ} = k \cdot \frac{A_{ABC}}{2}.$$

$$\text{If } MS \parallel QT \Rightarrow \frac{A_{BQT}}{A_{BMS}} = k^2 \Rightarrow A_{BQT} = k^2 \cdot \frac{A_{ABC}}{4}.$$

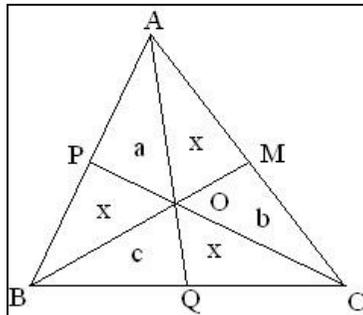
In this case we shall obtain  $4k^2 + 8k - 5 = 0 \Leftrightarrow (2k - 1)(2k + 5) = 0$  so we

shall have:  $k = \frac{1}{2}$ .



**Second problem:**

We shall note a point  $O$  inside of the triangle  $ABC$  and  $AO \cap BC = \{Q\}$ ,  $BO \cap AC = \{M\}$  și  $CO \cap AB = \{P\}$ . If the triangles  $AOM$ ,  $COQ$ ,  $BOP$  are equivalent, then the point  $O$  will be the center of gravity of the triangle  $ABC$ .



## Solution:

We shall consider  $A_{AOM} = A_{COQ} = A_{BOP} = x$  și  $A_{AOP} = a, A_{COM} = b, A_{BOQ} = c$ .

Applying the Ceva Theorem we shall obtain:  $\frac{AM}{MC} \cdot \frac{CQ}{QB} \cdot \frac{BP}{PA} = 1$  (\*)

$$\text{But: } \frac{x}{b} = \frac{AM}{MC}, \frac{x}{c} = \frac{CQ}{QB}, \frac{x}{a} = \frac{BP}{PA} \stackrel{(*)}{\Rightarrow} x^3 = abc. (1)$$

We shall obtain:

$$\frac{x}{b} = \frac{2x+a}{x+b+c} \Rightarrow x^2 + bx + cx = 2bx + ab.$$

$$\frac{x}{c} = \frac{2x+b}{x+a+c} \Rightarrow x^2 + ax + cx = 2cx + bc.$$

$$\frac{x}{a} = \frac{2x+c}{x+a+b} \Rightarrow x^2 + ax + bx = 2ax + ac.$$

Summing the three relations we shall obtain:  $3x^2 = ab + ac + bc. (2)$

Using the relations (1) and (2) we have:  $ab + ac + bc = 3\sqrt[3]{a^2b^2c^2} \Rightarrow$  applying the means inequality  $a = b = c \Rightarrow a = b = c = x \Rightarrow O$  will be the middle center of gravity of the ABC triangle.

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