Applications about representing solutions for inequalities systems using Geogebra

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ABSTRACT: Given a function f: D → R, any sentence open call equation of the form f (x)> 0 and f (x)≥ 0 and f (x) <0 and f (x) ≤ 0. Inequation is called algebraic (grade I, II, III etc.) or transcendent (irrational, trigonometric etc.). Solving any inequality lies in finding its true crowd included in the definition of the function. Representation of systems inequalities and inequalities can be achieved very easily and efficiently with Geogebra.
KEYWORDS: Geogebra, systems, representing, inequalities.

1 Applications for linear inequalities systems

A system of linear equations is the system of inequalities grade one formed. For a representation of this system consider the following applications:

1) A company imports components for assembling two models of personal computers: PC1 and PC2. Following the sale of a product PC1 company a profit of 50 um (currency units - RON, EUR, $ …) and the sale of a product PC2 company a profit of 40 um. The following week production available 150 hours for assembly. Assembling a lasting 3:00 PC1 and PC2 a lasting 5 hours.

The company has in stock only 20 monitors PC2, that can be assembled weekly maximum of 20 computers PC2. Total storage space is 30 m^2. A 0.8 m^2 occupy PC1 and PC2 occupies a 0.5 m^2.

The company management wants to establish production plan for the next week (ie PC1 and PC2 determine the number of computers that will be assembled so as to be maximum total profit.

Consider all built computers will be sold and the remaining resources (packaging, PC1 monitors, etc.) are available from the company.

To obtain useful mathematical model synthesizing data is as follows:

resources	consumption per unit		available
	PC_1	PC_2	
resource 1(R_1) assembly hours	3 hours	5 hours	150 hours
resource 2 (R_2) storage space (m^2)	0,8 m^2	0,5 m^2	30 m^2
unit profit (um)	50 um	40 um	

Steps to create the mathematical model are:

Step 1. Identify variables and measurement units. The decision variables are the unknowns' problem. The problem required production plan that is the number of each kind of computer that will assemble. So the decision variables (VD) are:

x_1 = the number of the computers PC1
x_2 = the number of computer PC2

which will assemble next week. With VD builds mathematical model.

Step 2. Expression total profit that must be maximized through a function called the objective function (FO), so the performance criterion. Because profit for PC1 is 50 um and the company produces a number x_1 of computers PC1, x_1 profit for all computers 50x1. Similarly, profit for x_2 all computers PC2 is 40x2.

So the objective function FO:

$$f(x) = 50x_1 + 40x_2 = MAX\,(um)$$

Step 3. Expression restrictions. Restrictions or constraints are conditions to be resources met conditions for manufacturing, selling, that expresses the conditions under which the study is conducted.

Restrictions:

• The restriction regarding assembly (resource 1) $3x_1 + 5x_2 \leq 150$ hours
• The restriction on storage (resource 2) $0.8x_1 + 0.5x_2 \leq 30$ m^2
• The restriction on the number of monitors PC2 (resource 3) which determines no. The PC2 that will assemble: $x_2 \leq 20$.

Step 4. Conditions for non-negativity. These variables are required for a decision, so because their interpretation (computers) and because finding the optimal solution method. $x_1 \geq 0$, $x_2 \geq 0$.

The set of admissible solutions (SA) is the set of coordinates (x_1, x_2) of all the points that satisfy all the conditions of non-negativity restrictions. The points are permissible in the area and outline them.

For the 3 restrictions so we can get representation:

Right d_1: $3x_1 + 5x_2 = 150$ passing through $A_1(50.0)$, $B_1(0.30)$.

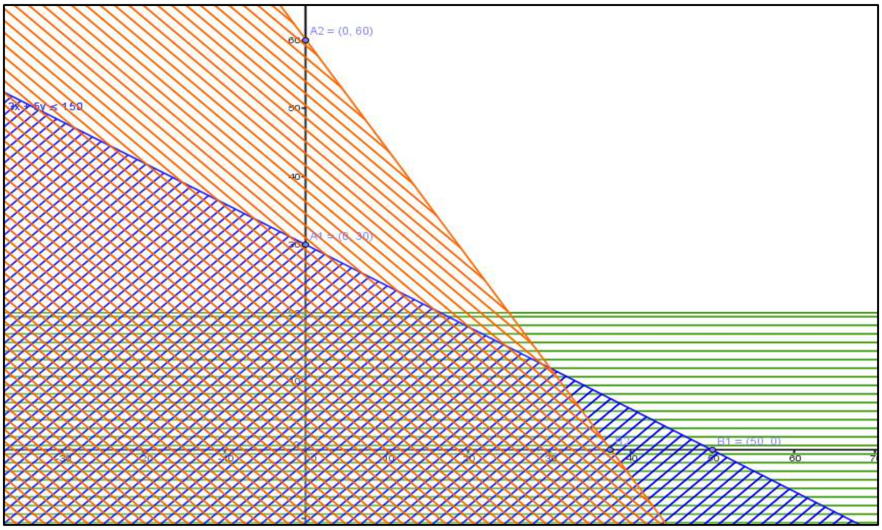
Inequation $3x_1 + 5x_2 < 150$ is satisfied in A_1OB_1 triangle as the origin O(0,0) satisfies $3 \cdot 0 + 5 \cdot 0 < 150$.

Right d_2: $0,8x_1 + 0,5x_2 = 30$ passing through $A_2(37.5;0)$, $B_2(0;60)$.

Inequation $0,8x_1 + 0,5x_2 < 30$ is satisfied triangle A_2OB_2.

Right d_3: $x_2 = 20$ is parallel to the OX_1.

Inequation $x_2 < 20$ is satisfied in the area of OX_1 and right axis $x_2 = 20$.



Shaded area is the allowable 3 colors. The area has an infinite number of points allowable, so the crowd is acceptable solutions in this case infinite.

In this lot we must choose the point whose coordinates gives the highest objective function value. That point will be the best solution. We clearly need to restrict the set of points where to look for the best solution so that this set is finite. The area is a convex admissible.

It searches only the tops of the crowds. Convex polygon coordinates peaks surrounding area permissible is set (x_1, x_2) admissible basic solutions. The optimal solution is the one of the tips of the polygon.

To determine the optimal solution can do so:

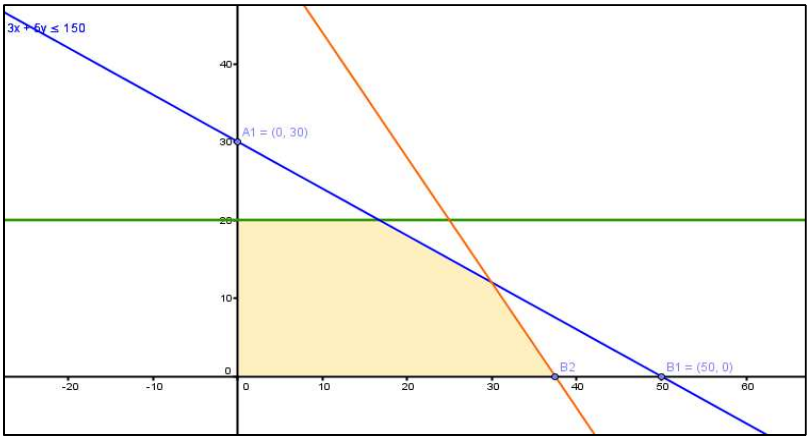
The calculated coordinates of the peak A, A_2, M, N, B_3, and then the objective function is the value of each peak. The optimal solution is the point (x_1, x_2) which gives the highest value target function.

We O(0,0), A_2(37,5;0), $\{M\} = d_1 \cap d_2$ so be solved system

$$\begin{cases} 3x_1 + 5x_2 = 150 \\ 0{,}8x_1 + 0{,}5x_2 = 30 \end{cases} => M(30{,}12)$$

$\{N\} = d_1 \cap d_3$ so be solved system

$$\begin{cases} 3x_1 + 5x_2 = 150 \\ x_2 = 20 \end{cases} => N(\frac{50}{3}, 20) \text{ and } B_3(0{,}20).$$



Objective function values at these points are:

$$\begin{cases} f(O) = 50 \cdot 0 + 40 \cdot 0 = 0 \\ f(A) = 50 \cdot 37{,}5 + 40 \cdot 0 = 1875 \\ f(M) = 50 \cdot 30 + 40 \cdot 12 = 1980 \\ f(N) = 50 \cdot \dfrac{50}{3} + 40 \cdot 20 = 1633{,}3 \\ f(B_3) = 50 \cdot 0 + 40 \cdot 20 = 800 \end{cases}$$

It follows that the best solution is the point M (30.12).
It will manufacture a number of

$$\begin{cases} x_1 = 30\ computer\ PC_1 \\ x_2 = 12\ computer\ PC_2 \end{cases}$$

and the maximum profit is 1980 um. This shows that the optimum solution for any other production plan can not get a higher profit than 1980 um.

2) An economic unit manufactures the products P_1, P_2 and P_3 using three resources: manpower, means of labor and raw materials. Specific consumption of each resource available quantities and selling prices of the products are given in the table below:

product / resources	P_1	P_2	P_3	available (physical units)
workforce	1	3	4	15
means work	2	5	1	10
raw materials	4	1	2	25
sale price (monetary	3	2	6	-

units)				

The mathematical model on which it is established optimal production program geared towards maximum production efficiency, has the form:

$$(\max) f = 3 \cdot x_1 + 2 \cdot x_2 + 6 \cdot x_3$$

$$\begin{cases} x_1 + 3x_2 + 4x_3 \le 15 \\ 2x_1 + 5x_2 + x_3 \le 10 \\ 4x_1 + x_2 + 2x_3 \le 25 \end{cases}$$

$$x_1, x_2, x_3 \ge 0$$

Using the rules of crossing the dual, dual problem arising following:

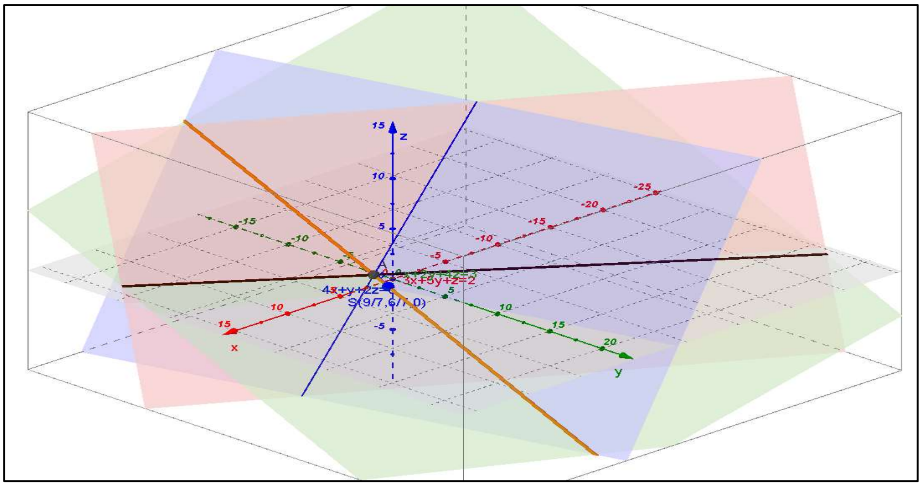
$$(\min) g = 15 \cdot u_1 + 10 \cdot u_2 + 25 \cdot u_3$$

$$\begin{cases} u_1 + 2u_2 + 4u_3 \ge 3 \\ 3u_1 + 5u_2 + u_3 \ge 2 \\ 4u_1 + u_2 + 2u_3 \ge 6 \end{cases}$$

$$u_1, u_2, u_3 \ge 0$$

After resolving the simplex algorithm to obtain the optimal solution to both problems last simplex table below:

			3	2	6	0	0	0
c_B	x_B	x_B	x_1	x_2	x_3	s_1	s_2	s_3
6	x_3	$\frac{20}{7}$	0	$\frac{1}{7}$	1	$\frac{2}{7}$	$-\frac{1}{7}$	0
3	x_1	$\frac{25}{7}$	1	$\frac{17}{7}$	0	$-\frac{1}{7}$	$\frac{4}{7}$	0
0	x_6	$\frac{35}{7}$	0	-9	0	0	-2	1
z_j	-	$\frac{195}{7}$	3	$\frac{57}{7}$	6	$\frac{9}{7}$	$\frac{6}{7}$	0
Δ_j	-	-	0	$\frac{43}{7}$	0	$\frac{9}{7}$	$\frac{6}{7}$	0



It may be a representation of this problem using GeoGebra, representing three planes, lines intersect plane, two by two, that crossing point between them.

2 Examples of irrational systems of equations

Inequation radical containing unknown into the equation is called irrational.
To solve irrational inequalities, usually necessary to prop up both members of the equation. Such transformations can make to non-equivalent to the initial inequalities and inequalities because of a multitude of solutions is in most cases an infinite, checking them is difficult. The only method that guarantees fairness answer is that, like the irrational solving inequalities are to be made only those changes that preserve their equivalent.

To represent such systems consider the application:

$$\sqrt{x^2+3x-18} > 2x+3$$

To solve the inequalities we consider the general case $\sqrt[2n]{f(x)} > g(x)$

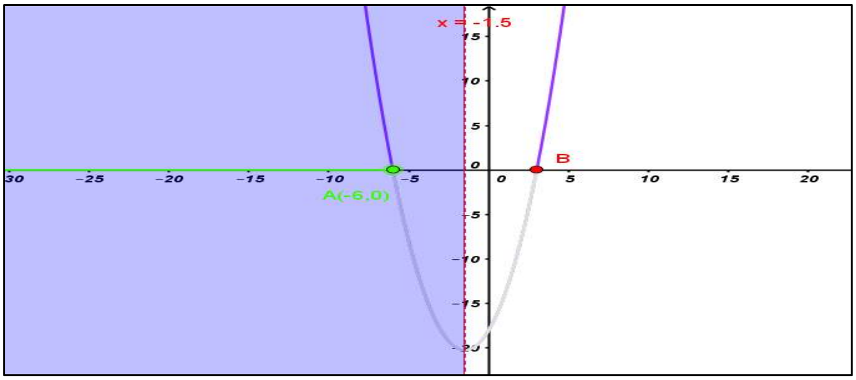
which is equivalent to the $$\left[\begin{array}{l}\begin{cases} g(x) < 0 \\ f(x) \geq 0 \end{cases} \\ \begin{cases} g(x) \geq 0 \\ f(x) > [g(x)]^{2n} \end{cases}\end{array}\right.$$

Under this approach, the problem studied, we

$$f(x) = x^2+3x-18, g(x) = 2x+3$$

$$\sqrt{x^2+3x-18} > 2x+3 \Leftrightarrow \left\{\begin{array}{l}\begin{cases}x^2+3x-18 \geq 0\\ 2x+3<0\end{cases}\\ \begin{cases}2x+3 \geq 0\\ x^2+3x-18>(2x+3)^2\end{cases}\end{array}\right. \Leftrightarrow$$

$$\begin{cases}x \in (-\infty,-6]\\ x \in \emptyset\end{cases} \Leftrightarrow x \in (-\infty,-6].$$



Representing the first equation is in the previous figure.

The second system has no solutions because equation

$$x^2+3x-18>(2x+3)^2 \Leftrightarrow x^2+3x+9<0$$

Attached equation $x^2+3x+9=0$ are $\Delta=-27<0$ and then $x^2+3x+9>0$. In this context $x \in \emptyset$.

3 Examples of exponential and logarithmic equations systems

An exponential equation is an inequation the unknown is superscript or phrase appears in the exponent.

A logarithmic equation is an equation in which the unknown appears as the base or argument or the argument expressions or logarithm base.

Exponential inequalities based on monotony properties of exponential and logarithmic functions. Thus, if the base is above par exponential and logarithmic functions are increasing and if the base is less exponential and logarithmic functions are decreasing and in the latter case equivalent inequation sign changes.

Systems of equations exponential and logarithmic functions are reduced after applying properties of logarithms operations with powers and systems of equations Grade 1 or 2. In addition, these systems shall be made for the conditions of existence and expressions bases in logarithms.

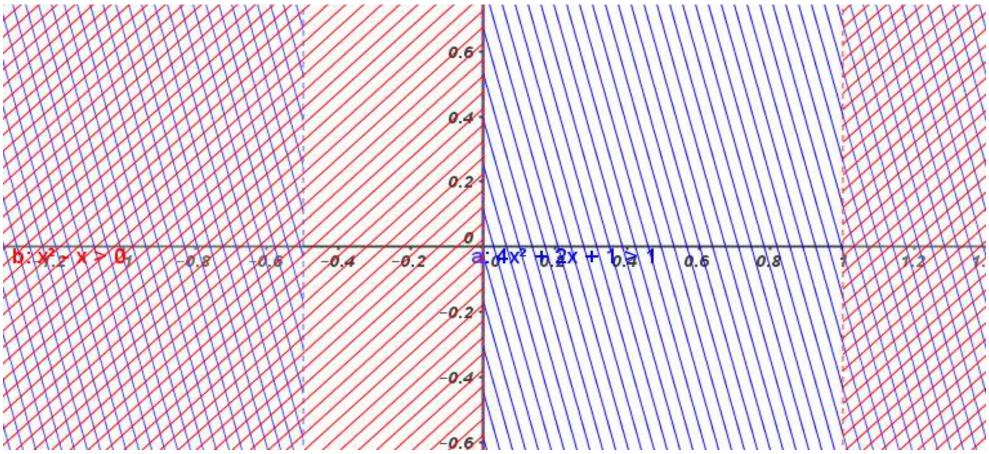
To solve exponential and logarithmic inequalities systems solutions resulting from the application properties of logarithms and operations with powers and equivalence of elementary operations have intersected with the conditions of existence for bases and expressions as logarithms.

To solve the inequation: $(4x^2+2x+1)^{x^2-x}>1$.
This can rewrite

$$(4x^2+2x+1)^{x^2-x}>(4x^2+2x+1)^0 \Leftrightarrow \left[\begin{array}{l}\begin{cases}4x^2+2x+1>1\\ x^2-x>0\end{cases}\\ \begin{cases}4x^2+2x+1<1\\ 4x^2+2x+1>0\\ x^2-x<0\end{cases}\end{array}\right.$$

$$\Leftrightarrow \left[\begin{array}{l}\begin{cases}\begin{cases}x>0\\ x<-\frac{1}{2}\end{cases}\\ \begin{cases}x>1\\ x<0\end{cases}\end{cases}\\ \begin{cases}x\in\left(-\frac{1}{2},0\right)\\ x\in\mathbb{R}\\ x\in(0,1)\end{cases}\end{array}\right. \Leftrightarrow \begin{cases}x\in\left(-\infty,-\frac{1}{2}\right)\cup(1,\infty)\\ x\in\emptyset\end{cases} \Leftrightarrow x\in(-\infty,-\frac{1}{2})\cup(1,\infty)$$

Decided on the final solution is the first system solution of inequalities, as in the following representation:

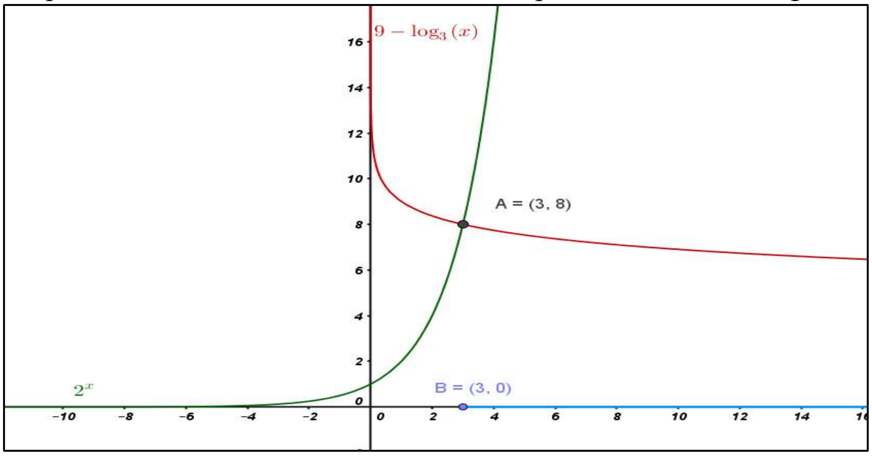


If we believe inequation system

$$\begin{cases}2^x\geq 9-\log_3 x\\ \log_5(x+2)\leq 4-x\end{cases}$$

To study another two inequations.

First, a member of the left equation is a strictly increasing, and the right function strictly decreasing and therefore the graphs of these functions can have at most one common point. The better we understand if their representation is envisaged.

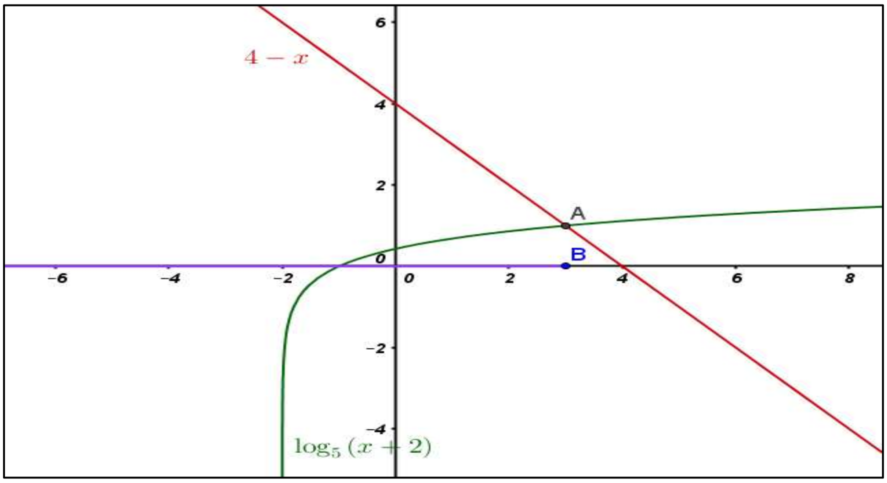


They have a single point of intersection (3.8). Check moreover follows:

$$2^3 = 9 - \log_3 3$$

In this context, the solution is the first inequation $x \in [3, \infty)$.

The second inequation is solved so similar



The solution to this is inequalities $x \in (-\infty, 3]$. The system solution is x=3.

Content of the work

1 Applications for linear inequalities systems

2 Examples of irrational systems of equations

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