Visualizing in polar coordinates with the Geogebra

Francisco Regis Vieira Alves
Instituto Federal de Educação, Ciência e Tecnologia do Estado do Ceará – IFCE.
Brazil
fregis@ifce.edu.br

ABSTRACT: In this article we discuss and describe some examples of application of the integrals in polar coordinates. We see that the software Geogebra provides means to describe complex regions in the plane. Moreover, explore and manipulate the graphic-geometric construction with the purpose to indicate the convenient definite integral. Finally, with some examples we show strategies of solution that not emphasize only the analytical standard approach in the teaching process of the integral Calculus.

KEYWORDS: Visualization, Polar Coordinates, Integral Calculus, Software Geogebra.

1 Introduction

Admittedly, the notion of polar coordinates has a prominent place in the teaching of Calculus in Brazil. While we recognize your relevance, we can not overlook some negatives elements. First, we observe that the hegemonic standard style of discussion by the book's authors is the analytical approach. Second, we must note that many complex constructions in polar coordinates become impossible without the computer's help.

In this perspective, we elaborated tree situations linked with the notion of polar coordinates and we demonstrate that description of one understanding's scenarios for the pupils is invaluable, when we do not use the technological instruments. Moreover, we can discuss the elements originated by the perception and visualization of the graphics provided by the

computer environment. In this way, we intend reduce the emphasis of the analytical and formal arguments in despite of final resolution in each problem situation.

The mathematical model is described by $A = 1/2 \int_{\theta_1}^{\theta_2} \rho(\theta)^2 d\theta$, where the function f is continuous and positive in the considered region of the plane and each point P is determined by a distance from a fixed point O and a angle θ from a fixed direction . So, we will show a way to extract and indicate the integrals directly from de graph provided by the software.

2 Visualizing regions in polar coordinates with Geogebra

We present some problem situations involved the notion of polar coordinates. We emphasize that these structured situation was thought in the way that promote the visualization and perception of the properties in the first stage of strategy's solution.

Situation I: Describe the region determined by the two curves in the fig.1 and indicate the definite integral corresponding.

Comments: We see in the fig. 1 the equations $\rho = 1$ and $\rho = 2\cos(2\theta)$. In the fig. 1-I we perceive that are intersection points determinate by these curves in the polar plane. However, we wish to describe some regions of the symmetry. So, we explore the construction (fig. 1-II) and we can identify two lines. Quickly, we determine the points from the conditions $2\cos(2\theta) = 1$.

Observing the fig. 1-II, we indicate the lines $y = x$ et $y = \left(\dfrac{\sqrt{3}}{3} \right) x$. We still observe that others regions can be obtained by the symmetry properties in the figure below. In any case, we will restrain our preliminary analysis to 1° quadrant of the polar plane.

In the first moment, we can study the properties of the symmetry in the fig 1. We must investigate with the manipulation of the construction in Geogebra with the purpose to indicate the convenient curves that corresponding to the colored regions that we constructed in the next figure. In the fig 2-II we colored, with the some basic functions of the software, four regions. We intend to determine the definite integrals of each one this.

Easily, regarding the biggest (green) region, we write $A = 1/2 \int_{0}^{\pi/6} 2\cos(2\theta) - 1 \ d\theta$ et, originating from the symmetry, we multiply this value for 2. In order to find the expressions for other integrals, we must extract correct information from de manipulation of the graph.
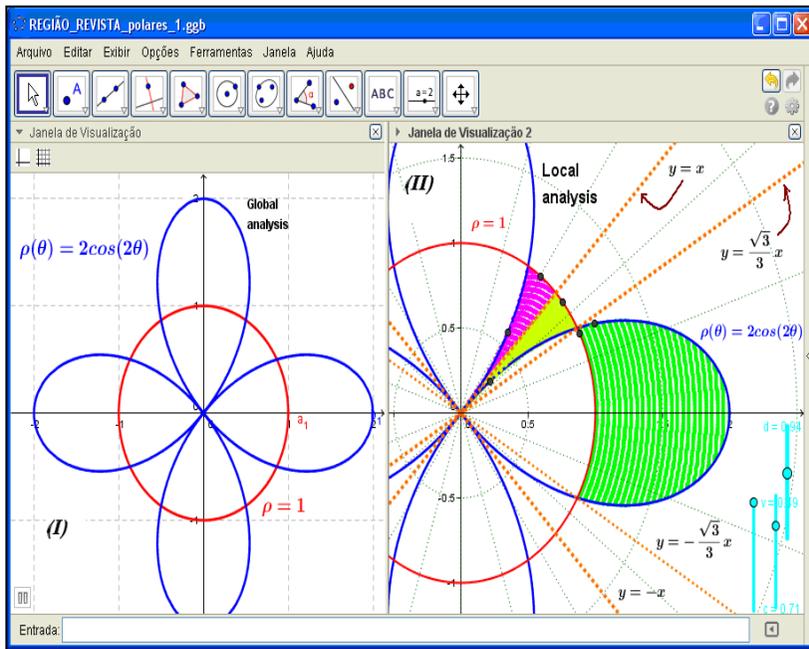
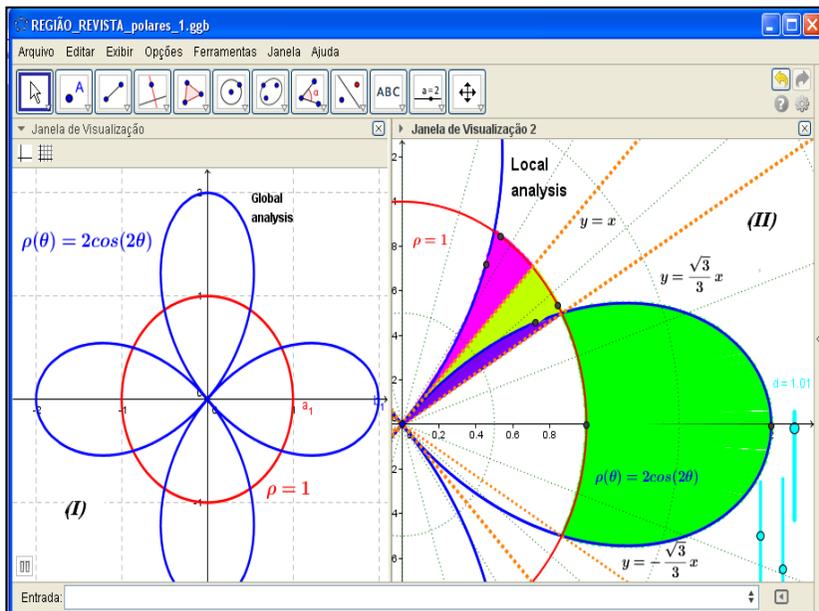**Fig. 1.** We identify the regions of symmetry with the help of the software Geogebra

Fig. 2. Discriminating the regions of the simmetry with the purpose to elaborate the polar coordinates integral

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In the fig. 2-II, we can write that $A = 1/2 \int_{\pi/6}^{\pi/4} 2\cos(2\theta) \ d\theta$ relatively the smallest region (purple) limited by these curves. On the other hand, with the aim to calculate the rose (and the light green) region indicated in the same figure, we will establish that $\dfrac{1}{2} \int_{\pi/6}^{\pi/4} 1 \ d\theta - \dfrac{1}{2} \int_{\pi/6}^{\pi/4} 2\cos(2\theta) \ d\theta$ correspondently to the light green region between the two lines. At the end, we can effectively determine the other region (in pink), from the symmetry again by multiplying by 2.

Indeed, we finalize with the expression $2 \times \left(\dfrac{1}{2} \int_{\pi/6}^{\pi/4} 1 \ d\theta - \dfrac{1}{2} \int_{\pi/6}^{\pi/4} 2\cos(2\theta) \ d\theta \right)$ correspondently the symmetry (fig. 2-II) that express the area of the pink and light green regions.

Situation II: Describe the region determined by the two curves in the fig.3 and indicate the definite integral corresponding.

Comments: We will consider the equations $\rho = 1$ and $\rho = 2\sqrt{\cos(2\theta)}$. In the first stage, we suggest the students realize a global visualization. We view in the fig 3-I the two considered curves. In these circumstances, we conclude that exist limited regions between these two plane curves. This scene allows producing the preliminary perceptions.
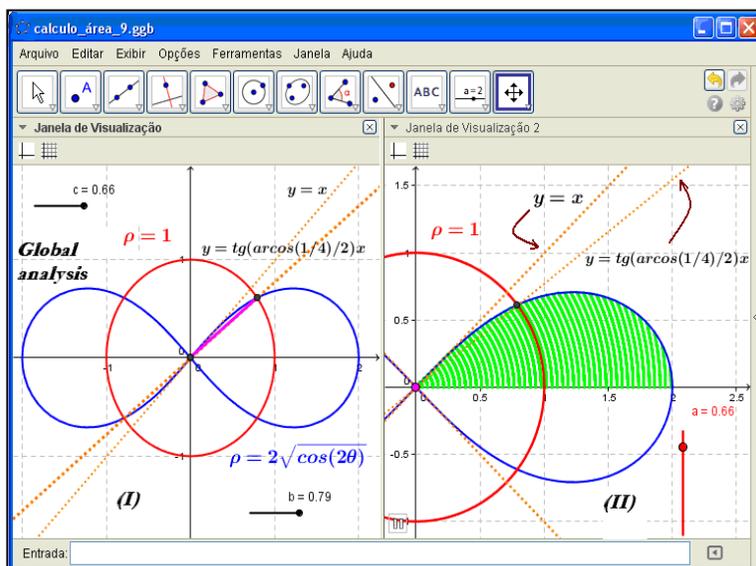

Fig. 3. The software allows explore the construction with the aim that find geometric and graph properties

In the fig. 4-II we can chose determinate the green region in the 1º quadrant. Again, we use the two lines indicated in the beginning (fig. 3) like references to determine the desired integral. So, from this region, we establish that $A = 1/2 \int_0^{\pi/4} 2\sqrt{2\cos(2\theta)} \; d\theta = \sqrt{2} \int_0^{\pi/4} \sqrt{\cos(2\theta)} d\theta$. If we choose and explore other regions, we can identify in the fig. 4-I an green region and an pink region in the plane. However, having a kind of awareness, we must conclude that we can't use one single integral. But, the pink region is described trivially by a expression $A = 1/2 \int_0^{(arc\cos(1/4/2))} 1 d\theta$ while the green region is obtained by $A = 1/2 \int_{(arc\cos(1/4/2))}^{\pi/4} 2\sqrt{2\cos(2\theta)} \; d\theta$.
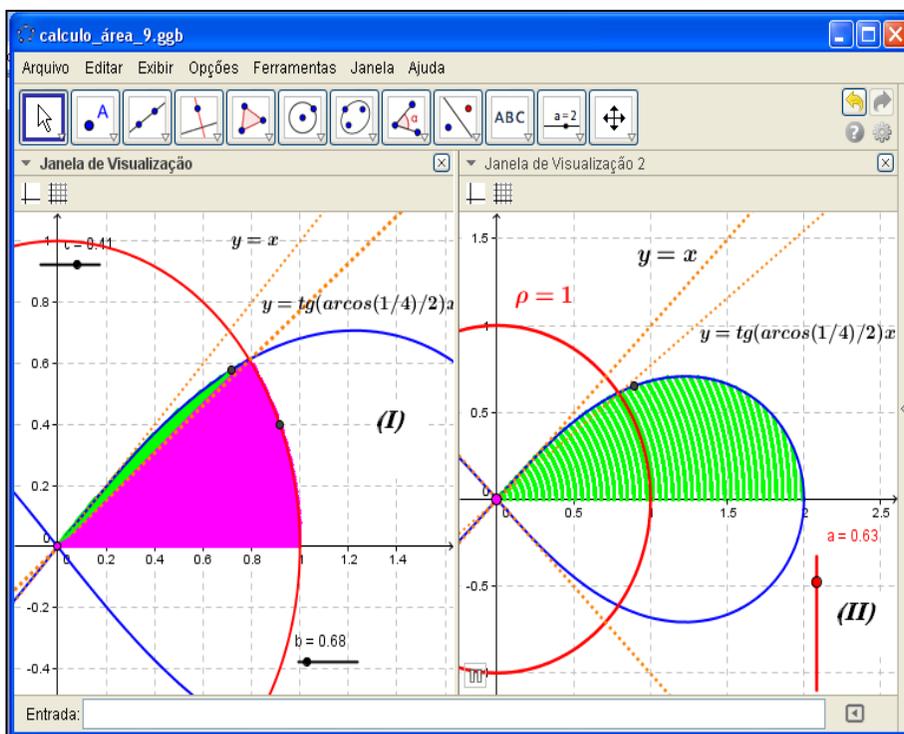

Fig, 4. We explore the properties of the construction elaborated in the Geogebra

We have to explain with more details this last integral. Indeed, we adopted the curves $\rho = 1$ and $\rho = 2\sqrt{\cos(2\theta)}$. We have a way to identify the behavior of intersection of these curves. With this purpose, we establish

$2\sqrt{\cos(2\theta)} = 1 \therefore \cos(2\theta) = \dfrac{1}{4}$. Finally, we find $\theta = \arccos(1/8)$. We notice that some basic commands of the software, like **Curva[s(t) cos(t), s(t) sen(t), t, 0, 6.28319]**.

We can color various regions in the scenario of the fig. 3. The next situation was structured in other to develop a precise analysis and elaborated perception directly to the graphs. Without the software, may be it seems difficult to elaborate such graphs. (see fig. 5-I)

Situation III: Describe the region determined by the two curves in the fig.5 and indicate the definite integral corresponding.

Comments: From a global analysis, we see the multiples intersection points between the two curves described by $\rho = 1 + sen(\theta)$ and $\rho = 2sen(3\theta)$. In the origin O, we acquire the visual sensation that exist a point with multiplicity. While, in others regions, only one intersection point. On the right side, we will develop a detailed analysis (see fig. 5-II).
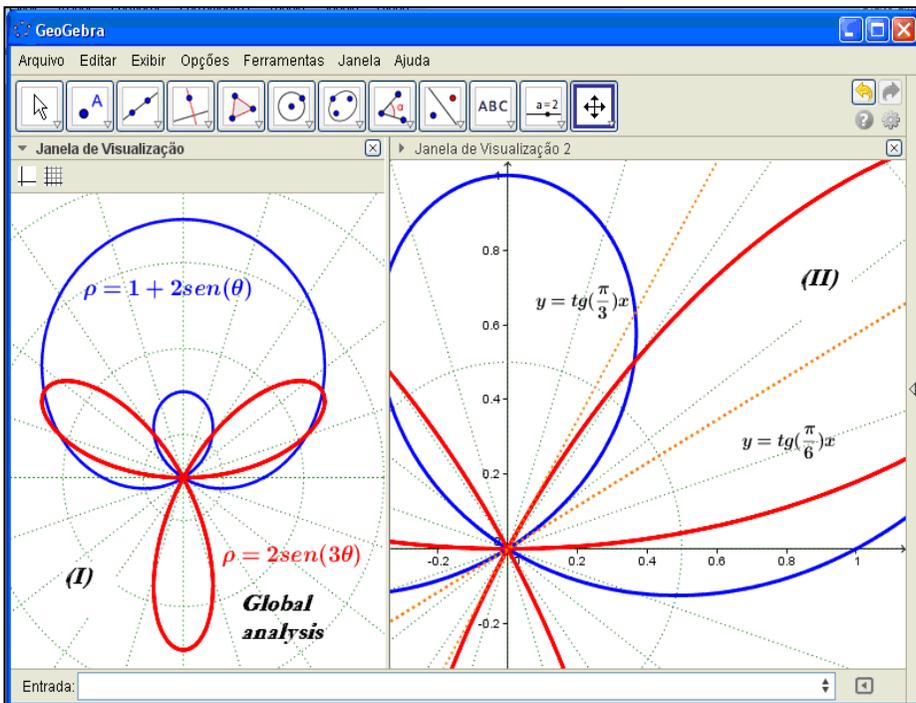

Fig. 5. We visualize and explore the curves in the polar plane with the aim to understanding the intersection points determined by the curves

After investigate and manipulate the construction, we start to discriminate too some analytical properties. With this goal, we consider $\rho = 1 + 2sen(\theta)$ and $\rho = 2sen(3\theta)$. We impose $1 + 2sen(\theta) = 2sen(2\theta + \theta) \leftrightarrow$ $1 + 2sen(\theta) = 2sen(2\theta) \cdot \cos(\theta) - 2sen(\theta) \cdot \cos(2\theta) \leftrightarrow 8sen^3(\theta) - 4sen(\theta) + 1 = 0$. Now, we take $t = sen(\theta) \therefore 8t^3 - 4t + 1 = 0$. We obtain with the software the following points $A = (-0.31; 0)$, $B = (0.31; 0)$ and $C = (0.5; 0)$.

With one of these points, we can determine dynamic points on the curves. We compare the fig. 5 and the fig. 6 with the goal to determinate and indicate the correct variation to the angles. Trivially, with the software, we find the real roots of the function $f(t) = 8t^3 - 4t + 1$. As we observe, all roots area real numbers, as indicated in the fig. 6-I.

We advert that sometimes, it's difficult to determine precisely the angles that we need to indicate the corresponding polar coordinates integral. In the fig. 6-II, we write two lines with to support our understanding. We record that the points A, B and C, can helps us to find the desired angles. In fact, we will establish $sen(\theta_1) = -0.31$, $sen(\theta_2) = 0.5$ and $sen(\theta_3) = 0.31$.

We use one of these values for to determine a line that divides the two regions with the last value indicated by θ_3. We perceive one green region (on the left side) and one pink region (on the right side).

A person can't be in a position to infer these properties without an computer environment. Moreover, we can explore and manipulate the construction with the goal that comprehends a way to determine the definite integrals.

Observing the fig 5 and 6, we determine the corresponding integral $A = 1/2 \int_{\theta_3}^{\pi/3} (2sen(3\theta)d\theta$. To complete our investigation, we can return to the fig. 5-I and conjecture the other regions where we could apply the same formulation and to find the corresponding area. We observe that with the selectors (in vertical), we can describe dynamic points over the two curves. This argument allows describe in color each interest region in plane.

The software permit explores a great diversity of situations and the teacher can take some moments for to provoke the scientific debate in the class. Moreover, we must observe that those problems situation can be solved by the use of double integrals.

Like a example in the fig 2-II we can substitute the $1/2 \int_{\pi/6}^{\pi/4} 1 \ d\theta - 1/2 \int_{\pi/6}^{\pi/4} 2\cos(2\theta) \ d\theta$ for only $\int_{\pi/6}^{\pi/4} \int_{2\cos(2\theta)}^{1} \rho d\theta d\theta$. Here we have used the 2 dimensional theorem of change of the variables, where $dxdy = \rho d\rho d\theta$.
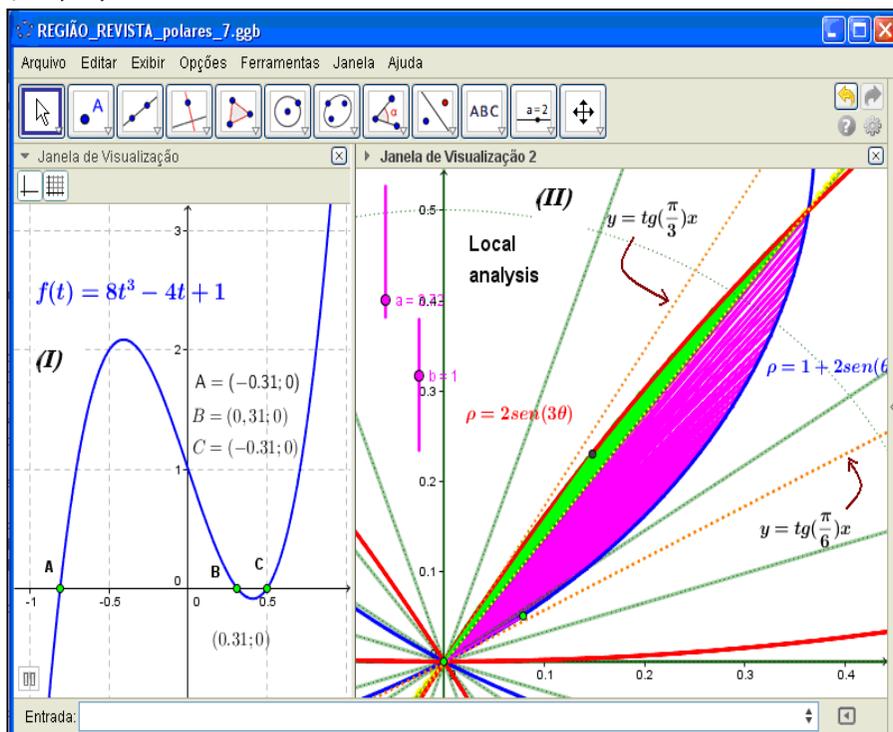

Fig. 6. With the zoom function of the software we can explore and identify the regions of symmetry and determine the corresponding integral in polar coordinates

Conclusion

We showed in this article that the software Geogebra promote the description of the integrals in polar coordinates directly from the graphic and specially the indication of the variation that concern a specific area. We detached the possibilities for comprehend the intersections points of two polar curves in the plane. However, when we seek this kind of point, we must know that one single point in Cartesian coordinates has multiple representations in polar coordinates. So, we suggest ally the perceptual properties extracted directly from the mathematical objet by intuition that constitutes our most powerful tool [MA02] with the analytical dates.

We showed with these situations that some complex region in the plane can be investigated with this software. By the use the moving points over the

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trajectories, we can color a specific region and after employ the analytical model for compare the final dates. Moreover, it is possible to solve these problems by double integrals. In such way, we can establish relevant conceptual relationships in the teaching of Calculus [FRV11].
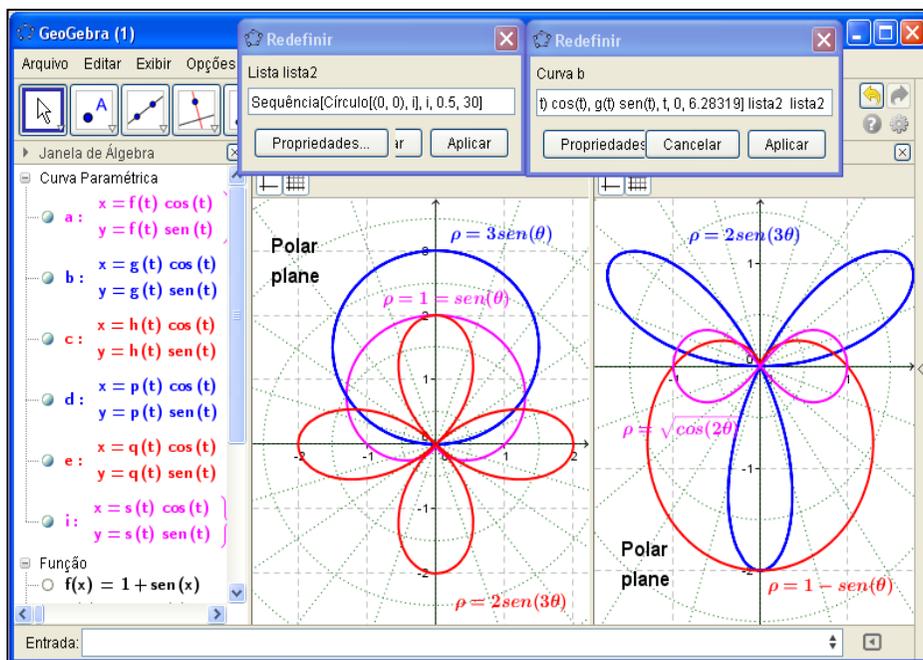

Fig. 7. Description of the software commands to construct the polar curves in the plane with the Geogebra

We assume a perspective that emphasizes the visualization and the perception as a way of to start as specific mathematical investigation for the students [BA10]. These proposed activities are the examples that we can avoid algorithmic routines in the teaching of the differential and Integral Calculus [FRV13]. Finally, in the teaching process, we can stimulate the students to formulate and conjecture yours strategies from the visualization, the perception and the understanding the graphic meaning of the integrals in polar coordinates. Furthermore, properties that changes when we use the transformation $(x, y) = (\rho \cos(\theta), \rho sen(\theta))$ (*). We indicate the possibilities of this mathematical transformation (*) in the computer environment. (see fig. 7).

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