Exploring the L´Hospital rule using Geogebra

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ABSTRACT: In this article we show that the exploration of the software Geogebra can change the analytical style of teaching this topic. Thus, through three situations, we describe how this software allows extracts data from the visualization. In this scene, we indicate and discuss the manifestation of the indeterminate forms and complex symbols like $0/0$, ∞/∞, $0^0, 1^\infty$, etc.

KEYWORDS: Visualization, L`Hospital rule, Differential Calculus, Software Geogebra.

1 Introduction

The L´hospital rule is studied in any book of Calculus. Its description allows to get rid of certain types of indeterminate forms. However, the authors of Calculus´ book in Brazil tend to emphasize only the procedural aspects and analytical arguments Thus, we show that using the software Geogebra, we can indicate and structured a way of interpreting symbols from de geometric-graphic forms.

Indeed, from the interpretation of the functions graphs, we can compare the numerical approximation of image, in the case that the variable ‘x’ tends or manifest certain behavior. Our proposal avoids the precipitated employment from the usual analytical rules. We observe too, that the software Geogebra can help the teacher and motivate the participation of the students, at the same time they visualize and can manipulate and explore the complex graphs of the functions in the computer environment.

2 Manifestation of undetermined forms

In this section, we discuss the manifestation of undetermined forms. The style used by the authors of the books in Brazil is characterized by the emphasis of the algebraic description of the some classes of limits.

In fact, we find symbols, like $\frac{0}{0},\frac{\infty}{\infty},0^0,0^\infty,1^\infty,+\infty-\infty$, etc. The problem is restricting the manner of teaching these symbols in academic environment only in the formal style. We accentuate that the statements below avoid to indicate one single mode or argument for the resolution. When write "?" in the limit below, we desire to leave to the student decide the point at which the rule must be check. Then, we see the first situation.

Situation I: Decide the nature of the limit indicated by $\lim_{x\to ?}\frac{\ln(1+sen^2(x))}{2x^2}$ and indicate a point which we have an indeterminate form.

Comments: Based on the graph to the left side, we conclude that $\frac{\ln(1+sen^2(x))}{2x^2}\to\frac{0}{0}$, for values $x\to 0^+$ (fig. 1-I). On the other hand, we see that $\frac{\ln(1+sen^2(x))}{2x^2}$ may exist, because the oscillation of the graph tends to decrease, for large values of the 'x'. Thus, it make sense to use L´Hospital rules indicated by the limits $\lim_{x\to 0^+}\frac{\ln(1+sen^2(x))}{2x^2}$ or $\lim_{x\to +\infty}\frac{\ln(1+sen^2(x))}{2x^2}$.

We observe that : $\frac{f'}{g'}=\frac{\frac{2sen(x)\cos(x)}{(1+sen^2(x))}}{4x}=\frac{sen(x)\cos(x)}{2x(1+sen^2(x))}$. We write that $\lim_{x\to 0^+}\frac{sen(x)\cos(x)}{2x(1+sen^2(x))}\to\frac{0}{0}$. Based on the figure below, we perceive that the indeterminate persists. In fact, we se from the graphic below that both f' e g' through the origin, however, we can't decide which one is closer to the point $(0,0)$ (fig. 1-I). We proceed our analyses with the following:

$$\frac{\left(sen(x)\cos(x)\right)'}{\left(2x(1+sen^2(x))\right)'}=\frac{\cos^2(x)-sen^2(x)}{2(1+sen^2(x))+4x\cdot sen(x)\cos(x)}\to\frac{1}{2}.$$

This value allows compare the analytical results with the assumptions that we produced, from a preliminary examination of the graphs. Arrows indicate the direction of the movement performed on the graph and the locus of the $0/0$ appears.

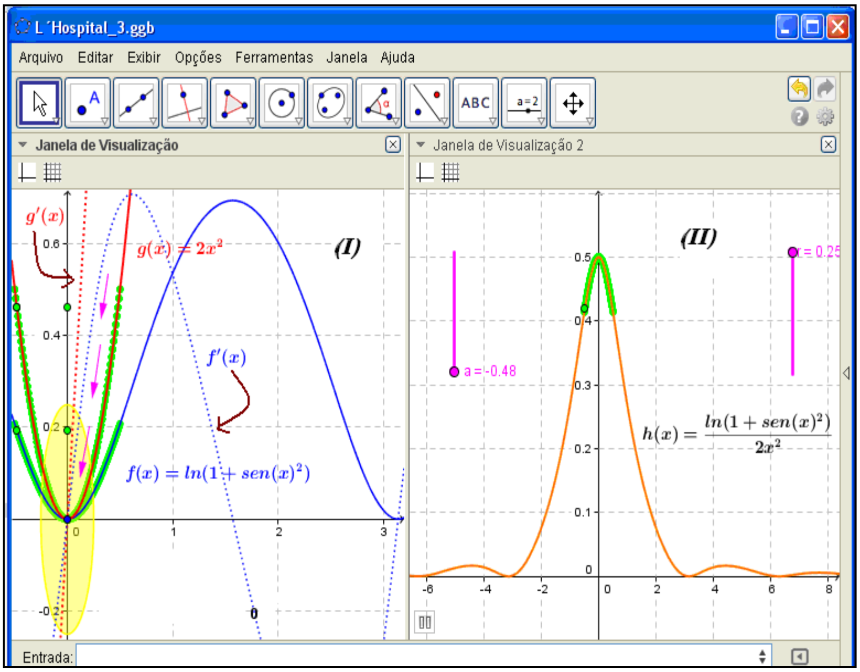

Fig. 1. Manifestation of an indeterminate form $0/0$ from a geometric-graphic frame

Situation II: Decide the nature of the limit indicated by $\lim_{x\to ?}(1+sen(4x))^{\cot g(x)}$ and indicate a point which we have an indeterminate form.

Comments:

In this case, we consider $h(x)=f(x)^{g(x)}=(1+sen(4x))^{\cot g(x)}$.

Based on the graph below, we see that the indeterminate form is the type $1^{+\infty}$ In fact, for the values $x\to 0^{+}$, we write that $(1+sen(4x))\to 1$ and $\cot g(x)\to +\infty$. So, we visualize the manifestation and the locus of an indeterminate form in the geometric-graphic nature.

We note that the conjectures produced by the visualization on the graph should be compared with the dates in analytical form. Arrows indicate the dynamic movement.

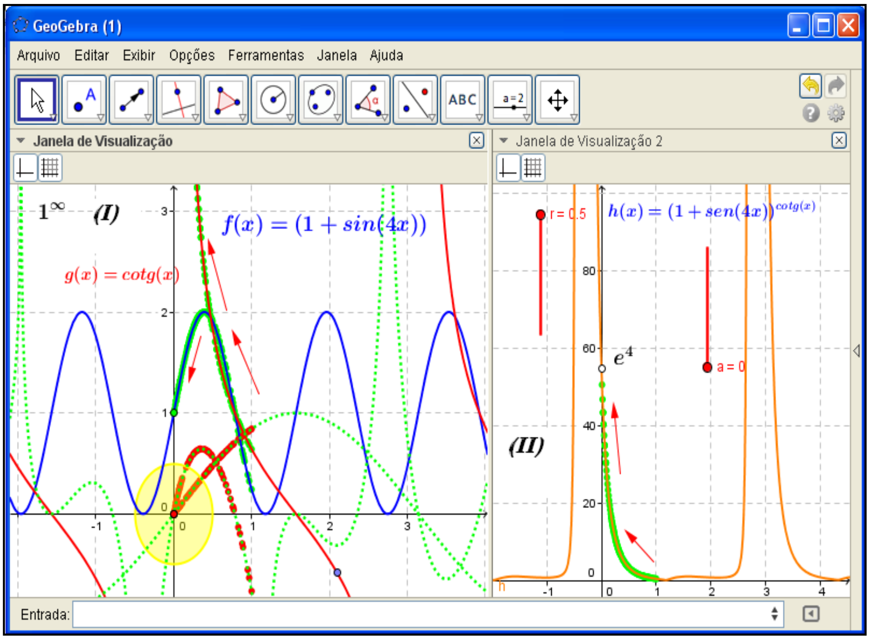

Fig. 2. Manifestation of an indeterminate form 1^∞ from a geometric-graphic frame

So, we write the following expression, by elementary propriety $(1+sen(4x))^{\cot g(x)} = e^{\frac{\cos(x)\cdot\ln\ 1+sen(4x)}{sen(x)}} = e^{p(x)}$. This argument is standard in the book´s Calculus in Brazil. We observe that the expression $p(x) = \dfrac{\cos(x)\cdot\ln(1+sen(4x))}{sen(x)} \overset{x\to 0^+}{\rightarrow} \dfrac{1\cdot(0)}{0}$. So, by this argument, we pass to another indeterminate form. We localize a neighborhood in origin $(0,0)$ and perceive another section of the graph. By fig. 2-II, we indicate and conjecture that the value expected is $e^{p(x)} \overset{x\to 0^+}{\rightarrow} e^4$.

Thus, by the L`Hospital rule, we write: $\dfrac{-sen(x)\cdot\ln(1+sen(4x))+\cos(x)\cdot\ \ 4\cos(4x)\ /(1+sen(4x)}{\cos(x)}$ and, en virtue that expression, we can deduce that: $\dfrac{-sen(x)\cdot\ln(1+sen(4x))+\cos(x)\cdot\ \ 4\cos(4x)\ /(1+sen(4x)}{\cos(x)} \overset{x\to 0^+}{\rightarrow} \dfrac{0+1\cdot\ 4}{1} = 4$.

Finally, we confront the dates observed in the beginning (fig. 2) of discussion with this dates in the analytical form.

Situation III: Decide the nature of the limit indicated by $\lim_{x \to ?} \frac{x^2 \cdot \cos\left(1/x\right)}{sen(x)}$ and indicate a point which we have an indeterminate form.

Comments: With the support in the fig. 3, we can predict that the type of the indeterminate form is like $0/0$. While that the limit indicated by (*) $\lim_{x \to 0} \left(x^2 \cdot \cos\left(1/x\right)\right) / sen(x)$ when 'x' tends to 0 exist (fig. 3-I). If we write the expression f'/g', like predict the rule, we can conjecture that the limit doesn't exist (fig. 3-II), because the oscillation of your image in axe Oy.

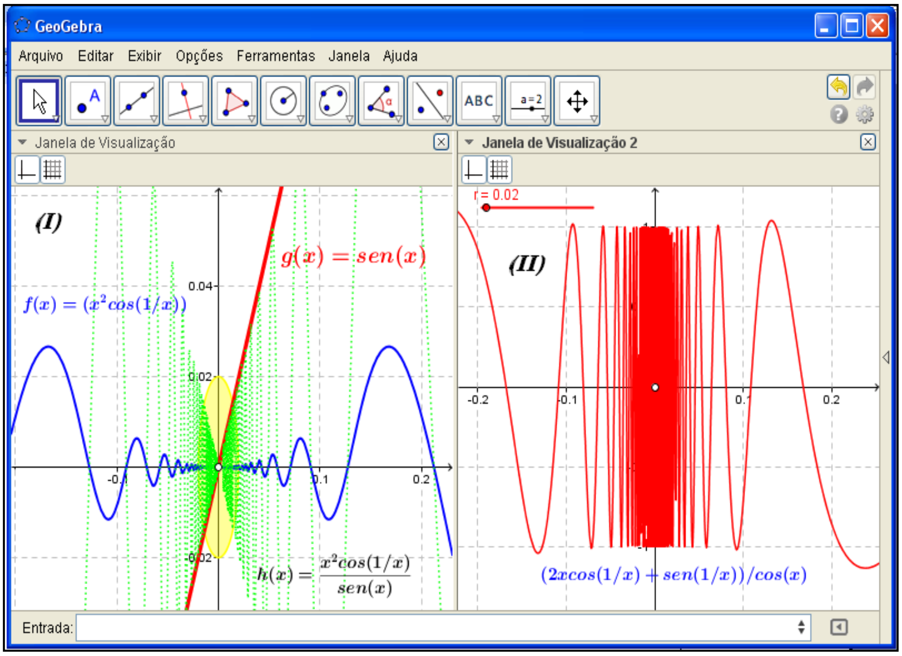

Fig. 3. With the help of the software, we can visualize the case where we can't use de L´Hospital rule and the locus of the indeterminate form manifested

In the last situation, we must write $\lim_{x \to 0} \frac{2x \cdot \cos\left(1/x\right) + sen\left(1/x\right)}{\cos(x)}$ and use another argument or analytical property. This class of situation can avoid the unconscious use of L`Hospital rule and alert the students. The students must understand that the limit (*) exist but, it should be respect all hypothesis that guarantee your application or not.

Conclusion

We showed that is possible to put in evidence the aspects graphics and geometrics related to the use of L´Hospital rule. We emphasized too the visualization as an element that produce the conjectures and describe an preliminary point of the teaching and learning [BCA13]. Furthermore, with the help of Geogebra, we solved the situations proposed by confront of thr analytical, graphics and numeric's dates.

We observe in the literature that the software Geogebra allows to make many applications of the use in advanced mathematical [TR13]. One of our specific concerns in this article was the signification of the complex symbols that we usually faced in the use of L`Hospital rule. A despite of the actions of the students when faced that rule, we can affirm that they apply quickly; however, we can't guarantee your understanding only in this fact.

On the other hand, the scenarios of visualization and exploration dynamic of this software can stimulate a tacit and intuitive sense of this classical rule, that arises from immediate perceptions and exploration of the graph [AFRV13]. Finally, as teachers, we can lead the students to view the geometric locus of manifestation of the complex symbols in Differential Calculus.

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