Comparative Geometry with Geogebra, Spherical Easel and Other Didactic Tools

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ABSTRACT: The proposal of simultaneous exploration of planar and spherical geometry on comparative way with using Geogebra, Spherical Easel and other didactic tools will be presented in this article. Comparative way means that we study different geometries at the same time, continuously comparing and contrasting one with the other. We begin with the plane and sphere, because these are familiar from school and everyday experience. The example of the lesson conducted with mentioned tools and comparative method will be presented.

KEYWORDS: planar geometry, geometry on the sphere, comparative method, constructivism

1. Introduction

We can draw and study various figures on many different surfaces, but school geometry only focuses on properties of the plane. Solids are mostly treated as three-dimensional objects, without paying any attention to constructions on their surfaces. At the same time, everyday experience offers a lot of opportunities to observe figures of different types, such as circles, triangles, quadrilaterals on the earth-globe, a model of the spherical surface. Do figures on the plane have the same properties as those on the sphere? Is geometry of the plane different from geometry of the sphere? These questions can best be studied by simultaneous application of well-chosen software materials and concrete models.

Application of software is most effective when used in tandem with other types of didactic tools. Experience shows that an adequate didactic situation inspires the student to act on his own, to construct knowledge required by the curriculum and beyond. To achieve this aim, the teacher should use relevant tools and methods, including discussion with students to help them reason and argue in a constructive manner.

2. Theoretical Background

Presented proposal is based on:

- theory of constructivism
- strategy of active education
- comparative method

Constructivism is a theory of learning and an approach to education that lays emphasis on the ways that people create meaning of the world through a series of individual constructs. It is a learning process which allows a student to experience an environment first-hand, thereby, giving the student reliable, trust-worthy knowledge. The student is required to act upon the environment to both acquire and test new knowledge.

Strategy of active education (acting learning) is the approach based on activities of students different from only watching, listening and making notes. It takes place in the classroom when students solve problems, perform experiments, do investigations, put hypotheses, verify these hypotheses – simply construct knowledge.

Comparative method means teaching and learning different approaches to the same subject in order to help understanding and creative thinking by comparison and contrast of different viewpoints, making use of materials carefully selected and structured in accordance with the needs of the given age and ability group.

3. Example

Topic: **Circle Circumscribed of a Triangle**

Type of school: Grammar school

Class time: 2 · 45 minutes

Aims:

- Deepening knowledge about properties of triangles on the plane,
- Acquisition of basic knowledge about another geometric system, spherical geometry, especially properties of triangles on a sphere,
- Forming skills in mathematical reasoning and proof, formulating hypotheses, and drawing conclusions,

- Improving skills to discuss and present arguments,- Forming skills to use didactic tools (software and models) in research work that leads to problem solving.

Methods of work at the lesson: problem solving, discussion, work with computer, exercises in constructions,

Forms of work at the lesson: common work and individual work,

Didactic tools: computer with software (GeoGebra and Spherical Easel), earth-globe, tools for constructions on the plane, models and tools for constructions on the sphere.

Lesson plan – the main stages:

- Defining a circumcircle by the teacher or by the students from Internet or other sources,
- Reasoning by students (with help of the teacher if needed) to establish a point as the center of a circumcircle of a planar triangle, - define the circle, and the perpendicular bisector of a segment, and determine a point equidistant from the three vertices of the triangle,
- Putting the key question: Is it possible to find a circumcircle to every triangle? In other terms: Do perpendicular bisectors of the sides of every triangle intersect in one point?
- Research work of students, using software GeoGebra to verify the hypothesis: Is it possible to find a circumscribed circle to each triangle?
- Formulating the conclusion,
- Constructing the circumscribed circle of a triangle- in program GeoGebra,- in exercise book, individually by each student,
- Discussing applications of acquired knowledge,
- Working on spherical surface (such as an earth-globe) and discussing usefulness of knowledge about circumscribed circle of a spherical triangle,
- Students set up a hypothesis about a circumcircle around a spherical triangle, and use software Spherical Easel to verify their hypothesis.
- Discussion: Can we accept the verification of our hypothesis in 3D geometry working on a 2D screen? What should we focus on while watching spatial constructions on the computer screen?
- Making construction of the circumcircle of a spherical triangle on a palpable model,
- Comparison of properties of geometrical figures on a plane and on a sphere in discussed scope,
- Final conclusions.

4. The lesson as it took place in Białystok, Poland, June 2012:

The lesson on this topic was conducted in the Second Public Middle School in Białystok with 20 students aged 15. Invited teacher: Anna Rybak.

The equipment of the classroom: one computer with proper software, smart board, tools for constructions on a plane, models and tools for constructions on a sphere (working in groups), and one earth-globe.

Steps of the lesson:

- Discussion with drawings on blackboard, hypothesis about a circumcircle around a planar triangle.
  Teacher's remark: It was quite difficult for students to formulate hypothesis about the position of a center of circumcircle around a planar triangle. How to find a point equidistant from all vertices of a triangle. Students couldn't invent anything, so the problem was reduced to the simpler one: how to find a point equidistant from two points? Discovery: perpendicular bisector of a segment! Now, step by step, hypothesis was formulated: perpendicular bisectors of sides of a triangle intersect in one point.

- Work with Geogebra on smartboard, verification of hypothesis, making construction:

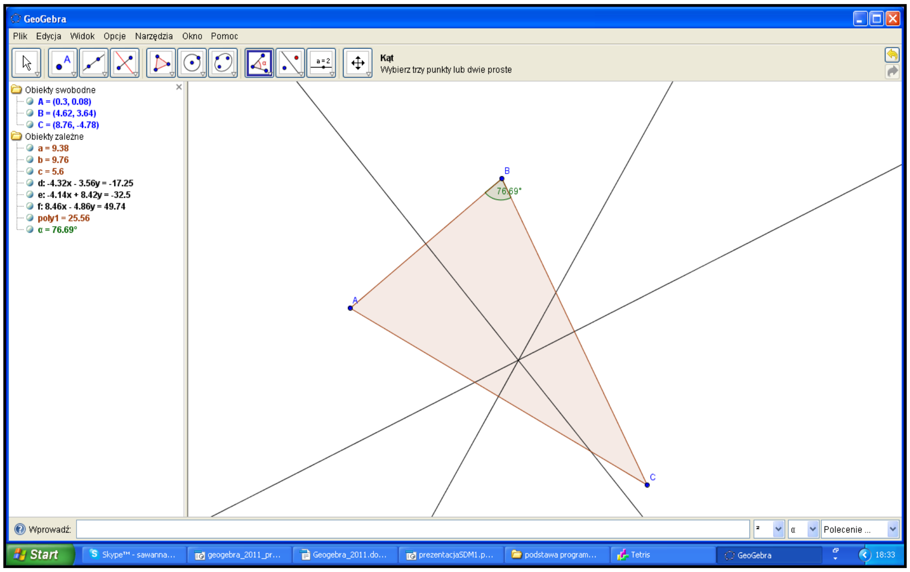

Fig. 1. Perpendicular bisectors of sides in acute triangle intersect in one point. This point is located inside the triangle.

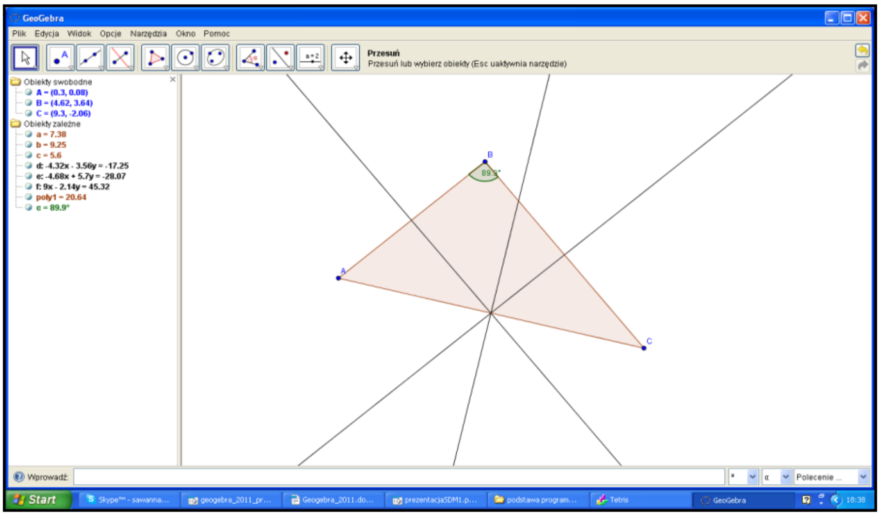

Fig. 2. Perpendicular bisectors of sides in right triangle intersect in one point. This point is located on hypotenuse.

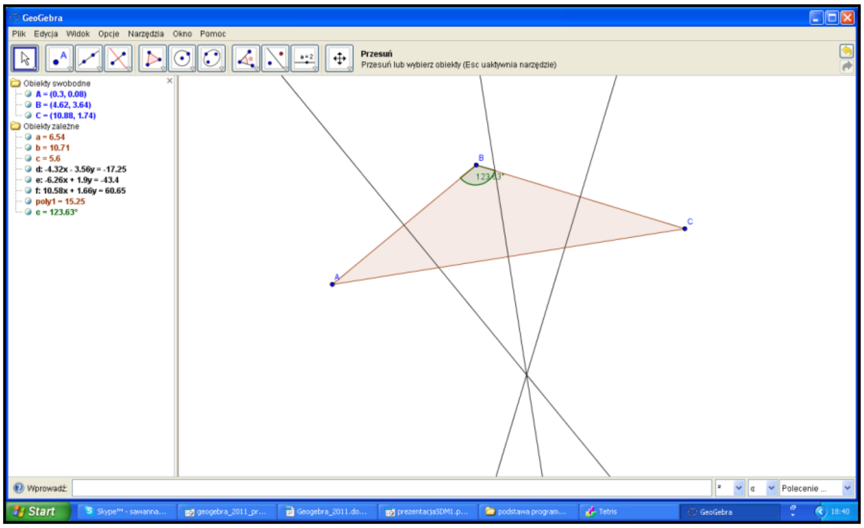

Fig. 3. Perpendicular bisectors of sides in obtuse triangle intersect in one point. This point is located outside the triangle.

Conclusion from work with Geogebra: Perpendicular bisectors of sides in each triangle intersect in one point. This point is the center of circle circumscribed about a triangle. The main conclusion of this stage of work is: For each triangle it is possible to construct circle circumscribed about it.

- Introductory discussion about spherical geometry using earth-globe and models:

Fig. 4. Presentation of a teacher. Construction of the circle circumscribed of a triangle on plane is visible at the smartboard.

- Set up a hypothesis about circumscribing a circle around a triangle on a sphere:
  Teacher's remark: Students were absolutely not used to consider figures on a spherical surface. They considered a sphere as a solid and probably first time “saw” some figures on its surface (especially earth globe was useful in this part of discussion). Students understood that some properties of figures on a sphere are different from properties of figures on a plane (for example triangle with three right angles exists on a sphere, does not exist on a plane), so were very careful with formulating hypothesis. After longer discussion they said: It is possible to circumscribe a circle about a spherical triangle.

- Work with Spherical Easel on smart board, verification of hypothesis:

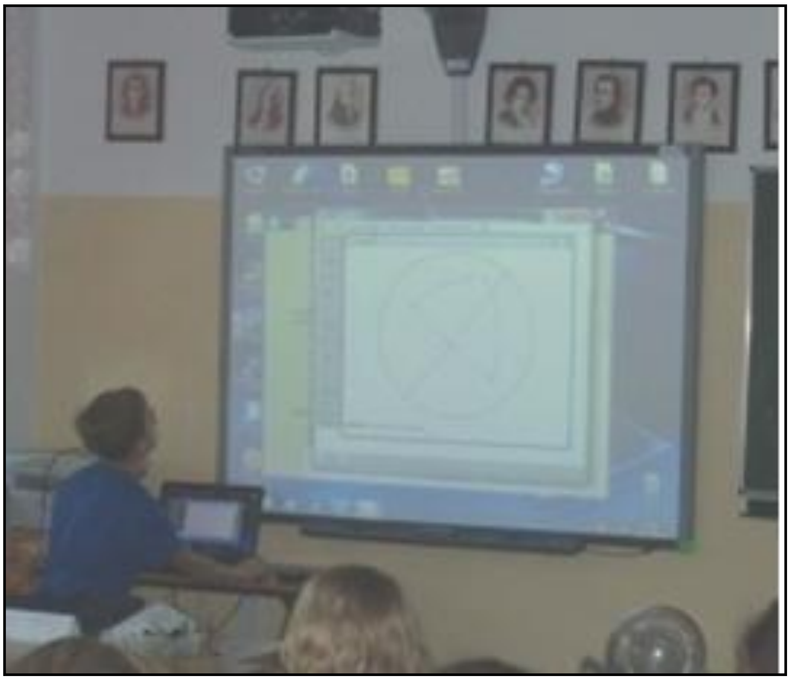

Fig. 5. Student makes construction in Spherical Easel. Spherical triangle and perpendicular bisector of one side are visible.

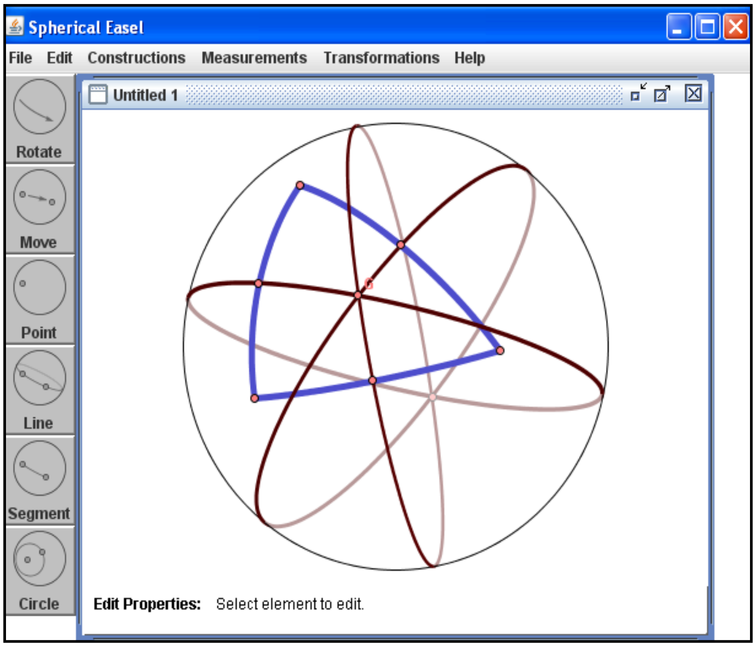

Fig. 6. Perpendicular bisectors of sides in acute spherical triangle intersect in two points. One of them is located inside the triangle.

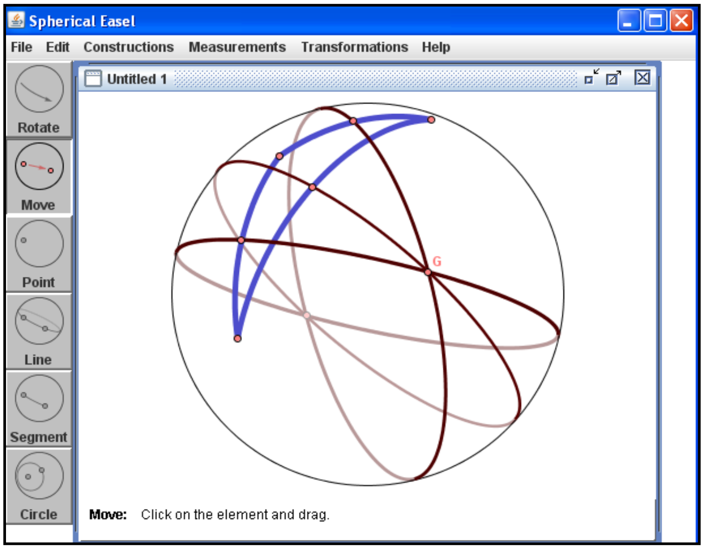

Fig. 7. Perpendicular bisectors of sides in obtuse spherical triangle intersect in two points. Both of them are located inside the triangle.

Conclusion from work with Spherical Easel: Students understood very quickly that spherical straight lines intersect in two points. They are familiar with this fact from work with earthglobe. So their conclusion was: It is possible to circumscribe a circle about each spherical triangle.

- Discussion “by the way”: Can we always accept all images from 3D geometry working on a 2D screen? What should we focus on while watching spatial constructions on the computer screen? What is our conclusion from the observation of the screen:

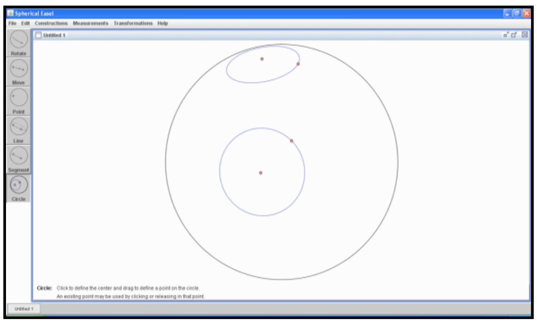

Fig. 8. Distortions of a 3D shape (spherical circle) dependent on a location of the figures on 2D screen

Conclusion: It is useful to complete work on two dimensional screens with work on three dimensional models.

- Work with models in groups, making constructions:

Fig. 9. Students make constructions on the models

Fig. 10. The result of students' work: circle circumscribed on spherical triangle.

- Final conclusions: It is possible to circumscribe a circle about a triangle on a plane and on a sphere. The point of intersection of perpendicular bisectors of sides of a triangle is a center of this circle. Students found some practical applications of this knowledge in both cases: on a plane and on a sphere. Teacher's remark: At the end of the lesson students said: “We never thought in this manner about mathematics, about geometry, about figures. It is so interesting that geometry on a sphere exists. Will you come to us for another similar lesson?"

5. Conclusions

Using computer programs in education about 3D geometry should be supported with using another didactic tools. Well-planned activities based on software GeoGebra and in some cases on Spherical Easel inspire students to investigate and construct mathematical knowledge on their own. Using comparative method gives opportunity to enhance students' knowledge and expand their mathematical thinking.

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