Abstract: Content and Language Integrated Learning (CLIL) can support the using of minority language in different subjects. In our paper we present this method in the case of school mathematics. First, we describe the process of gaining knowledge in teaching mathematics. We will present some students’ works of future Slovak minority teachers. These activities are possible to make also in GeoGebra. Our activities are oriented to grid paper and tangrams.

Keywords: Computer Based Math Education, Motivation in Mathematics education, Content and Language Integrated Learning (CLIL), GeoGebra activities with grid paper and tangrams.

CLIL - Integrated teaching of minority languages and non-language subjects

The motivation for the implementation of CLIL - an integrated teaching of minority languages and non-language subjects are European Commission's recommendations, particularly the Office of Commissioner for Education and Culture. European Commissioner Ján Figel 2006 states:

„Multilingualism is at the very heart of European identity, since languages are a fundamental aspect of the cultural identity of every European. For this reason, multilingualism is referred to specifically – for the first time – in the brief of a Commissioner. I am honoured to be that Commissioner. “(CLIL, 2006)
CLIL (Content and Language Integrated Learning) is an educational method for teaching non-language subjects through a minority language. It's an innovative approach that changes ways in which students are introduced to the curriculum, and that accelerates the acquisition of basic communication skills in a minority language.

Project results and comparisons show that this way of learning accelerates and increases the quality of teaching in general - educational and vocational subjects, as well as language training. CLIL strikes a balance between language and vocational training. Non-language subject is developed through a minority language and minority language by non-language subject. Minority language is used as an educational tool, not only as the result of teaching.

The objective this method of teaching is to improve the abilities and skills of students in minority language by the language as a tool of communication and not as a separate subject. CLIL can be considered as an educational method by which the EU promotes the linguistic diversity, and has a positive impact on language learning.

**Why CLIL?**

This teaching method has several advantages within minority schools. At this point we want to mention a few.

- When teaching with CLIL method, the aim is on the particular activity and not the minority language itself.
- This approach provides the opportunity to learn to think in that language and not only learn the language as such. CLIL allows students to practice the minority language in learning another subject.
- CLIL is the opportunity for graduates to develop their skills using foreign or minority languages and therefore to increase their personal potential for an advantageous position in the labour market.
- The curriculum can be explained first in Hungarian and later extended in the Slovak language, or vice versa.
- The activities in both languages should be complementary.

From advantages that CLIL brings, we can mention following ones:

- overall improvement of student communication skills in a minority language,
- deepen awareness of the minority language, official language and other languages
- Increased student motivation through real educational situations in the teaching of minority languages
- increase the fluency of expression, a wider range of vocabulary
- active involvement in lessons,
- a positive attitude towards the minority language,
- development of own national and cultural awareness
- preparation for practical life and work in a multicultural society.
- CLIL provides opportunities that allow students to use a minority language naturally, in such a way that they gradually forget about the use of minority languages and focus only on content.
- In the CLIL method the minority language is associated with other objects. In the classroom there are two main goals: one is the subject, topic, and another one is the language.
- This is the reason why CLIL is sometimes called a dual-focused teaching.
- CLIL can really do a lot, increases the willingness, wanting and ability to learn both - language minority and non-language subject.

The process of gaining knowledge as a sequence of five stages

Model of the process of gaining knowledge in mathematics education is based on five stages (Hejný et al., 2006, p. 15). It starts with motivation and its cores are two mental lifts: the first leads from concrete knowledge to generic knowledge and the second from generic to abstract knowledge. The permanent part of the gaining of knowledge process is crystallisation, i.e. inserting new knowledge into the already existing mathematical structure.

The whole process can be described by a scheme.

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abstract knowledge  →  crystallisation
↑ abstraction
generic model(s)
↑ generalisation
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Motivation is the tension which occurs in a person’s mind as a result of the discrepancy between the existing and desired states of knowledge. The discrepancy comes from the difference between “I do not know” and “I need to know”, or “I cannot do that” and “I want to be able to do that”, sometimes from other needs and discrepancies, too. For example, in the parking lot, there are two cars and three more will come, how many cars there will be? Isolated models - models of a new piece of knowledge come into mind gradually and have a long-term perspective. For instance, the concepts of fraction, negative number, straight line, congruency or limit develop over many years at a preparatory level. For our example we can use concrete objects from the real life for example two yellow apples and three red apples, two chairs and three other chairs and so on.

Generic model in the scheme of the process of gaining knowledge is placed over the isolated models indicating its greater universality. The generic model is created from the community of its isolated models and represents these models. For example fingers or bullets on counter represents chairs, apples and other objects.

Abstract knowledge gives birth to abstract knowledge. It’s a deeper view into that knowledge. New knowledge, relationships, concepts and dependencies between objects...
are defined and give independence. A student at this stage is verifying new knowledge on used model.

Crystallisation is the phase, in which the pupil after its entrance into the cognitive structure, a new piece of knowledge begins to look for relationships with the existing knowledge. If the pupil understand for example $2+3 = 5$, there is easy to find through models, that $5 – 2 = 3$ or $5 – 3 = 2$.

Automation is after mentioned five phases. In this stage we try calculate with pupils without models. The fact that a student answers quickly, correctly and with confidence does not imply that his/her answer is based on the appropriate image. For instance, the pupil knows that $2 \times 4 = 8$ but he/she cannot answer how much he/she has to pay for 2 lollypops costing 4 crowns each or what $3 \times 4$ is without going back to the beginning of the 4 times table. His/her knowledge is burdened with formalism; by this we mean the characteristic feature of mechanical knowledge. There is in this case very important the using of isolated and generic models. If teacher find by the pupil formalism or non-understanding of some notion, it is important to return to of isolated and generic models. The teachers often make the mistake that they didn’t use these models and by pupils’ problems they haven’t the possibility to return to some models.

**CLIL method in school mathematics and GeoGebra**

This method is applicable also in mathematics both in the preparation of future minority teachers of mathematics and also in Slovak primary and secondary schools in Hungary. The existence of a bilingual grammar is also an inspiration for new teaching methods of application of the Slovak language in the educational process at minority school in Hungary. According to Beardsmore (2008) the results of CLIL researches show that the monolinguals seemed to be stronger in their acquisition of knowledge of facts, whereas the bilinguals were better in acquiring the mathematical operations. In other words, the research revealed a difference between informational knowledge and operational knowledge for the two groups of subjects. Informational knowledge refers to the capacity to memorise, or « knowing that », whereas operational knowledge refers to the capacity to apply what one knows to new circumstances, or « knowing how ». Operational knowledge is significant for creativity, whereas formational knowledge serves more as a tool on which creativity must be built up. The studies on the learning of mathematics in a bilingual context were confirmed amongst different school populations, both in primary and secondary education, and even amongst beginners in second language programmes.

The study Domínguez (2011) shows one example the using CLIL method in mathematics teaching with text tasks. This teaching was in English and Spanish:
Tu maestra de arte te dio 3 paquetes de papel construcción para que hagan banderitas de México. Un paquete es de hojas verdes, uno es de hojas blancas, y uno es de hojas rojas. ¿Cómo podrías hacer 60 banderas de México?

In our case we try to make some CLIL activities with future Slovak minority teachers at Faculty of Education in Szarvas, Hungary. We used the figures prepared from tangram parts. These parts are possible to prepare in GeoGebra.

We have following plane figures – pictures of animals:
We try to make them in GeoGebra. The following picture shows two animals from Figure 2:

First we analyze the tangram parts from mathematical point of view and find their names in Slovak and Hungarian language and later we try to formulate sentences in both languages (Slovak and Hungarian).

<table>
<thead>
<tr>
<th>Slovak</th>
<th>Hungarian</th>
<th>English</th>
</tr>
</thead>
<tbody>
<tr>
<td>Štvorec</td>
<td>Háromszög</td>
<td>Square</td>
</tr>
<tr>
<td>Trojuholník</td>
<td>Egyenlő szárú háromszög</td>
<td>Triangle</td>
</tr>
<tr>
<td>Rovnobežník</td>
<td>Paralelogramma</td>
<td>A rectangle</td>
</tr>
<tr>
<td>Rovnoramenný trojuholník</td>
<td>Derékszögű háromszög</td>
<td>Isosceles triangle</td>
</tr>
<tr>
<td>Pravouhý trojuholník</td>
<td>Téglalap</td>
<td>Right triangle</td>
</tr>
<tr>
<td>Obdĺžnik</td>
<td>Ház</td>
<td>Rectangle</td>
</tr>
<tr>
<td>Komín</td>
<td>Kémény</td>
<td>Chimney</td>
</tr>
<tr>
<td>Pes</td>
<td>Kutya</td>
<td>Dog</td>
</tr>
</tbody>
</table>

Table 1 Figures and tangram parts
Zajac beží pred domom.
Pes naháňa zajaca.
Ryby sú v akváriu.

A nyúl fut a ház előtt.
Kutya kergeti a nyuszit.
A halak akváriumban vannak.

Rabbit is running in front of the house.
A dog is chasing a rabbit.
The fish are in the aquarium.

Table 2 Sentences

These activities support not only the building of mathematical notions by the students, but they have possibility to express their knowledge in Slovak and Hunagarian language and develop their communicative abilities. According Beardsmore (2008) bilingual children have a greater faculty for creative thinking at their disposal. They perform significantly better in tasks which require not the finding of the single correct answer to a question, but where they are asked to imagine a number of possible correct answers, for example, giving the maximum number of interesting and unusual uses for a cup.

The activities with future Slovak minority teachers can prepare them for bilingual work with pupils. The mathematics serves it not only for Slovak language teaching, but also for developing of pupils activities. Such examples are possible to find by Billlich (2008), Tkačik (2007) and Gazdíková & Mišút (2007). Work with GeoGebra allows using the methods of cooperative learning (see Jablonský (2006)).

**Conclusion**

The GeoGebra research community has already international character which brings the opportunity to exchange the experiences in the field of motivation of pupils and students in Mathematics education. We describe in our article some possibility to use multilingual character of the GeoGebra software. There is suitable for this kind of activities the eTwinning initiative. This is a European action that promotes school collaboration and networking through the use of ICT between schools in Europe. GeoGebra has the link at the webpage


The Faculty of Education of Catholic University in Ružomberok is a partner in the network CEEPUS HU 28 Active Methods of Teaching and Learning of Mathematics and Informatics coordinated by University of Miskolc (see Körtesi (2009). GeoGebra is useful also for teachers of some natural sciences, for this reason it is interesting to make research in the interdisciplinary dialogue between Mathematics education and education of natural sciences (see English version of the webpage http://oddid.ku.sk).
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